

Interface motion in a random medium: mean field theory

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

1992 J. Phys. A: Math. Gen. 25 L555

(<http://iopscience.iop.org/0305-4470/25/9/009>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 171.66.16.62

The article was downloaded on 01/06/2010 at 18:27

Please note that [terms and conditions apply](#).

LETTER TO THE EDITOR

Interface motion in a random medium: mean field theory

Heiko Leschhorn

Theoretische Physik III, Ruhr-Universität Bochum, D-4630 Bochum, Federal Republic of Germany

Received 24 December 1991

Abstract. The motion of an interface in a random medium is studied by a stochastic differential equation, with terms corresponding to an external driving field, interface elasticity, and a (quenched) random background field. For driving fields F smaller than a threshold field F_c the interface is pinned, i.e. the velocity $v = 0$. F_c and v are calculated within a discretized mean field theory. For F close to the threshold field we find that v grows linearly with $F - F_c$. Simulations of the mean field equations are in agreement with the analytical results.

Spin models in random fields are good representations of many impure materials. For example strongly anisotropic diluted antiferromagnets in a uniform field can be described by the random field Ising model [1, 2]. When these systems are quenched to low temperatures, they can be either in a stable long-range ordered state or in a metastable microdomain configuration, depending on the cooling conditions. The domain structure relaxes slowly, if at all, towards equilibrium, because of pinning of domain walls to random fields (RFs) or random bonds [2-4]. Experimentally, the relaxation process was investigated by measuring the magnetization [2]. For large domains in Ising-like models a part of a domain wall can be regarded as a planar interface because its curvature is small. Overhangs are neglected, so that the position of the interface is given by a single-valued function $u(x, t)$, where x is a D -dimensional position vector parallel to the interface. We want to investigate the slow motion of the interface driven by a constant field F . Treating the interface as an elastic membrane, one can write down the following equation of motion [3, 5, 6]

$$\partial u / \partial t = F + \tilde{\gamma} \nabla^2 u + g \eta(x, u) \quad (1)$$

where $\tilde{\gamma}$ is the interface stiffness, $\eta(x, u)$ is the local random field to be taken at the position of the interface, and g is a coupling constant. (In random bond systems $\eta(x, u)$ is the derivative $\delta V / \delta u$ of a random potential $V(x, u)$ with a short range correlation.) The RF variables η are independently distributed in space with zero mean. As an example one can choose a Gaussian distribution. For sufficiently large g the interface will become rough. Equation (1) may have a broader significance: Koplik and Levine introduced it to describe immiscible-fluid displacement in random media, as occurs when water is forced by a pressure gradient into an oil filled porous medium (e.g. sandstone rock) [5]. Recently, Kessler, Levine and Tu [7] simulated (1) to explain fluid displacement experiments in porous media of Rubio *et al* [8]. A similar equation has been proposed to describe impurity pinning of sliding charge-density waves (CDWs), [9] in which case the noise term $g\eta(x, u)$ in (1) is replaced by $h(x) \sin[u(x, t) - \beta(x)]$, where u is a phase, $h(x)$ is a random amplitude and $\beta(x)$ is a random phase.

We are interested in the mean velocity $v \equiv d\langle u \rangle / dt$ of the interface, where $\langle \dots \rangle$ denotes the average over x . In the thermodynamic limit and for $t \rightarrow \infty$ v is expected to be a time independent constant. We will consider the case $T = 0$ (T is the temperature), for which there is a sharp transition from a pinned interface ($v = 0$) to a moving interface ($v > 0$) at a threshold field F_c . (For $T > 0$ and $F < F_c$ one observes a very slow motion with a creep velocity calculated in [4].) The velocity v can be calculated by perturbation theory for weak disorder and $F \gg F_c$ [5, 6]. Here, the depinning behaviour $F \gtrsim F_c$ will be investigated, which can be considered as a dynamic critical phenomenon [9]. We expect $v \sim (F - F_c)^\theta$ near the threshold field F_c . It is the aim of the present letter to calculate v and F_c in a mean field model with an appropriate discretized geometry. For small F_c and v analytical results are obtained, which are in agreement with simulations of the mean field equation. The velocity v grows linearly with $F - F_c$, i.e. $\theta = 1$.

Let us consider a discretized version of (1) with a three-state RF distribution $\eta = 0, \pm 1$. The shortest interval in u -direction, over which the RF is constant, is denoted by a . The transverse coordinates x are considered as lying on a lattice with spacing a_{\parallel} (figure 1). A cell is assigned the value $\eta = +1$ (or -1) with a probability $q/2$, and $\eta = 0$ with a probability $1 - q$. First the case $q = 1$ will be considered. We call a cell with $\eta = +1$ (-1) a 'plus-cell' ('minus-cell'). For the part of the interface which is in one cell we use the term 'interface element'.

The velocity can be obtained by averaging (1):

$$v = F + g\langle \eta(x, u) \rangle \quad (2)$$

i.e. the problem consists in computing $\langle \eta(x, u) \rangle$, which in general depends itself on v . When computing the average over x in (2) (see figure 1) the value of $\langle \eta(x, u) \rangle$ will depend on the probabilities p_{\pm} to find an interface element on a plus- or minus-cell. In general, $p_- > p_+$ and $\langle \eta(x, u) \rangle < 0$. This is because the interface tends to spend more time on RFs with negative η than on positive RFs. For large v the interface at each RF will advance quite smoothly and the times t_{\pm} , which the interface elements spend on plus- or minus-cells, are almost independent of the sign of η . Thus, $p_+ \approx p_-$, $\langle \eta(x, u) \rangle \rightarrow 0$ and $v \rightarrow F$. As F is lowered towards F_c , the motion of the interface will become more and more jerky and $p_- > p_+$, $v \rightarrow 0$.

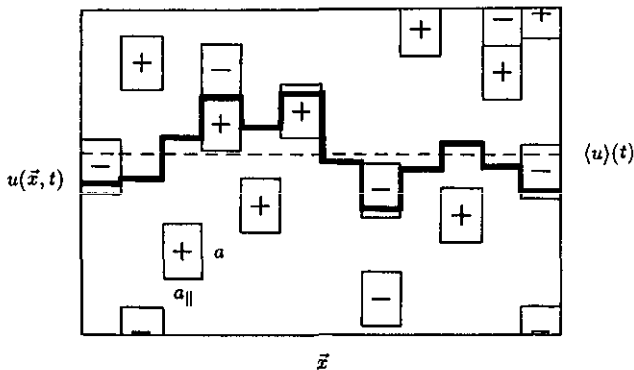


Figure 1. The discretized model. Cells in a given column are arranged on top of each other but the relative position in u -direction of different columns is assumed to be random. The \pm signs refer to the sign of RF-values $\eta(x, u)$. Cells with $\eta = 0$ are not drawn. All cells have a fixed width a_{\parallel} and a fixed height a .

Next the mean field theory will be described. The discretized Laplacian is given by $(\tilde{\gamma}/a_{\parallel}^2) \sum_{i=-D}^D [u(x+a_{\parallel}e_i, t) - u(x, t)]$. The mean field approximation consists in replacing the Laplacian in (1) by $\gamma[\langle u \rangle(t) - u(x, t)]$ with $\gamma = 2D\tilde{\gamma}/a_{\parallel}^2$:

$$\partial u / \partial t = F + \gamma(\langle u \rangle - u) + g\eta(x, u). \tag{3}$$

The mean field theory should be a good approximation for interfaces with bounded fluctuations and high dimensions D or long-range interactions. This kind of replacement was introduced by Koplik and Levine for equation (1) [5] and by Fisher for CDWs [9], where for the latter the exponent θ was found to be equal to $\frac{3}{2}$. Computer simulations of the equation of motion for CDWs with the discretized Laplacian give $\theta = 0.95 \pm 0.05$ for $D=2$ and $\theta = 1.16 \pm 0.04$ for $D=3$ [10]. (Note that for CDWs D is the dimension of the full space.)

Since all interface positions $u(x, t)$ are only coupled to a 'mean field' $\langle u \rangle(t)$, (3) can be regarded as independent of x , and one can reformulate the problem as follows. We consider the motion on a line with RFS of a single particle (interface element) which is coupled by a spring to a second particle ($\langle u \rangle$) which moves at a constant velocity v to be determined self-consistently. The average over x can be replaced by an average over the time t . Denoting by $\langle t_{\pm} \rangle$ the mean value of the times t_{\pm} averaged over a long run, the probabilities p_{\pm} are proportional to $\langle t_{\pm} \rangle$. In order to compute the times $\langle t_{\pm} \rangle$ we consider the motion of an interface element through one cell and solve the equation of motion (3) for plus- and minus-cells separately: with the initial conditions $t=0$, $u(t=0)=0$, $\langle u \rangle(t=0) = u^0$ the solution is

$$u = vt + (1 - e^{-\gamma t})[u^0 + (F \pm g - v)/\gamma] \tag{4}$$

and $\langle u \rangle(t) = vt + u^0$. The times $t(u=a) = t_{\pm}$ vary from cell to cell because the initial value of the distance u^0 between the interface element u and the average interface $\langle u \rangle$ can be different, whereas the parameters F , $\pm g$, γ , a , and v are equal for all cells. Therefore we have to average over the distribution $w(u^0)$ to obtain $\langle t_{\pm} \rangle$. For sufficiently large γ and a there exists a time interval $t_{\pm} \gg t \gg 1/\gamma$ for which one can neglect the exponential term in (4). Then, independent of u^0 ,

$$\langle u \rangle(t = t_{\pm}) - a = (v - F \mp g)/\gamma \tag{5}$$

which is u^0 for the next cell. Therefore the distribution $w(u^0)$ is just a sum of two delta-functions, depending on the sign of η of the previous cell. When an interface element is leaving a cell, there are four possible situations; as an example we demonstrate the case when an interface element on a plus-cell attempts to move into a minus-cell. Here, the distance u^0 is given by the upper sign of the RHS of (5). For small fields F the interface element first stays at the boundary between the two cells as long as $F - g + \gamma\langle u \rangle < 0$ (equation (3)). It will be dragged into the second cell when the moving average interface reaches $\langle u \rangle = (g - F)/\gamma$, which is the 'new' u^0 . The time, which the average interface $\langle u \rangle$ needs to move from the 'old' u^0 (equation (5)) to the 'new' u^0 , is called waiting time $t_0 = (2g - v)/(\gamma v)$. From (4) one obtains $t_{\pm}^{(+)} = (a + v/\gamma)/v$, where the sign in parentheses refers to the sign of the previous cell. During the waiting time t_0 the value of $\eta = \eta_0$ is not well defined, but we know that the velocity of the interface element is equal to zero. Thus, we subtract $0 = p_0[F + g\eta_0 + \gamma(\langle u \rangle - u)_0]$ from the RHS of (2), which takes the form

$$v = (p_+ + p_-)F + (p_+ - p_-)g - \gamma p_0(\langle u \rangle - u)_0 \tag{6}$$

where $(\langle u \rangle - u)_0$ is taken during the waiting time t_0 , $p_0 \propto \langle t_0 \rangle$ and $p_+ + p_- + p_0 = 1$. In a similar way as described above one obtains $t_{\pm}^{(-)} = t_{\pm}^{(+)} = a/v$ and $t_{\pm}^{(+)} = (a - 2g/\gamma)/v$.

Inserting all possible t_{\pm} in $\gamma t_{\pm} \gg 1$, we see that the neglect of the exponential term in equation (4) is justified for $a\gamma \gg v + 2g$. To calculate the probabilities p_{\pm} and p_0 one has to average t_{\pm} and t_0 over the two signs of the previous cell. Equation (6) becomes a quadratic equation for v [11]. The generalization of the calculation to the case $q < 1$ is straightforward and as the result we obtain

$$\frac{v}{g} = Q - \sqrt{Q^2 - \left(\frac{F - F_c}{F_c}\right) / \left(1 - \frac{3}{4}q\right)} \tag{7}$$

with $Q = [2 - q + 2n/q] / [2 - 3q/2]$ and $n = (a\gamma)/g$. The threshold field is

$$F_c = qg^2 / (2a\gamma) = qg / (2n). \tag{8}$$

For $[(F - F_c)/F_c] / [1 - \frac{3}{4}q] \ll Q^2$ equation (7) can be written as

$$\frac{v}{g} \approx \frac{F - F_c}{g + F_c(2 - q)}. \tag{9}$$

Thus the exponent θ is equal to one.

We have carried out computer simulations of the discretized mean field equation (3) for small fields $F_c < F < g$ to check equation (7) (figures 2, 3). The mean position

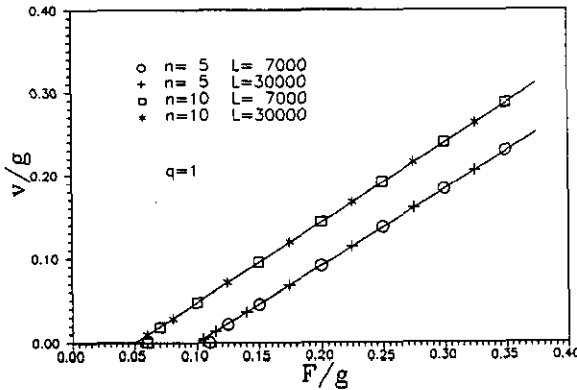


Figure 2. Velocity of the interface versus driving field: simulation of (3) and the resulting (7) (full curves) for $q = 1$. The statistical uncertainties are smaller than the size of the symbols.

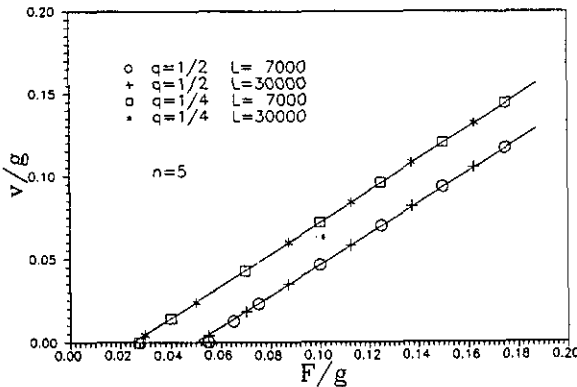


Figure 3. Velocity versus driving field: simulation of (3) and the resulting (7) (full curves) for $q < 1$.

$\langle u \rangle(t)$ is obtained by solving (3) numerically for L interface elements at the same time (see figure 1). For both cases $q = 1$ (figure 2) and $q < 1$ (figure 3) the numerical data are in agreement with the analytical result: for $n = 5$ the measured velocities $v(F)$ coincide with (7) within 0.7%, whereas for $q = 1$, $n = 10$ (20) the deviations are smaller than 0.4% (0.2%) except for F very close to F_c when the fluctuations of v become comparable to v itself. Similar to the effect of the random term $\eta(x, u)$ the fluctuations of v lower the effective velocity. Since the fluctuations are due to finite size effects we can diminish the deviations for F very close to F_c by enlarging L (figures 2, 3). As expected, the deviations increase with decreasing n . For $n = 2$ they reach a few per cent and for $n = 1$, (7) is certainly not valid.

To summarize, we have studied an equation of motion for an interface in a random medium with a discretized mean field theory. For small v and F_c the parameter of the equation of motion enter the results (7) and (8) in a simple way and the onset of the motion is linear. The analytical results are supported by a simulation of the mean field equation.

I wish to thank T Nattermann for introducing me into the problem and L-H Tang for very helpful discussions and a critical reading of the manuscript. This work was supported by a DFG grant through SFB 166.

References

- [1] Nattermann T and Villain J 1988 *Phase Trans.* **11** 5
- [2] Leitão U, Kleemann W and Ferreira I 1989 *Phys. Rev. B* **38** 4765
Pollak P, Kleemann W and Belanger D P 1988 *Phys. Rev. B* **38** 4773
- [3] Bruinsma R and Aeppli G 1984 *Phys. Rev. Lett.* **52** 1547
- [4] Nattermann T, Shapir Y and Vilfan I 1990 *Phys. Rev. B* **42** 8577
- [5] Koplik J and Levine H 1985 *Phys. Rev. B* **32** 280
- [6] Larkin A I and Ovchinnikov Y N 1974 *Sov. Phys.-JETP* **38** 854
Feigelman M V 1983 *Sov. Phys.-JETP* **58** 1076
- [7] Kessler D, Levine H and Tu Y 1991 *Phys. Rev. A* **43** 4551
- [8] Rubio M A, Edwards C A, Dougherty A and Gollub J P 1989 *Phys. Rev. Lett.* **63** 1685
- [9] Fisher D S 1985 *Phys. Rev. B* **31** 1396
- [10] Sibani P and Littlewood P B 1990 *Phys. Rev. Lett.* **64** 1305
- [11] Leschhorn H 1991 *J. Magn. Magn. Mat.* in press